Eleven dimensional superstring with new supersymmetry and $D = 10$ type IIA Green–Schwarz superstring

A.A. Deriglazov*

Department of Mathematical Physics,
Tomsk Polytechnical University, 634004 Tomsk, Russia

Abstract

A covariant action for closed $D = 11$ superstring with local κ-symmetry and global supersymmetry transformations obeying the algebra $\{Q_\alpha, Q_\beta\} = C\Gamma^{\mu\nu}P_\mu n_\nu$ is suggested. Physical sector variables of the model and their dynamics exactly coincide with those of the $D = 10$ type IIA Green–Schwarz superstring. It is shown that action of the $D = 10$ type IIA Green–Schwarz superstring can be considered as a partially gauge fixed version of the $D = 11$ superstring action.

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1 Introduction

While type IIA string and p-brane dualities (see [1, 2] and references therein) indicate on a possibility of M-theory unification in (10,1) dimensions [3, 4], theories which do not admit a direct M-theory unification may

*deriglaz@phys.tsu.tomsk.su

arise from F-theory in (10,2) dimensions [5, 6]. Motivated by the development of the F-theory, the authors of the recent works [7–13] have suggested a number of various models in a space with signature $(D-2,2)$. An interesting point is that they are based not on the super Poincaré algebra but on some other one, with commutator of supersymmetry generators of the type

$$\{Q_\alpha, Q_\beta\} \sim \Gamma^{\mu\nu} P_\mu n_\nu. \tag{1}$$

In particular, for the case of superparticle (superstring) models the algebra of such a type can be realized in a superspace as follows:

$$\delta\theta = \epsilon, \qquad \delta x^\mu = i\bar{\epsilon}\Gamma^{\mu\nu} n_\nu \theta. \tag{2}$$

To find interpretation for the vector n^μ, it was suggested to consider a system with two superparticles [7, 12, 13]. Then P^μ and n^μ may be regarded as momenta for each member of the system.

In this letter another interpretation of the algebra (1) and the vector n^μ will be presented for the case of $D = 11$ space with standard signature (10,1). Namely, we suggest a Poincaré invariant action for $D = 11$ superstring which is invariant under "new supersymmetry" [7–9] transformations (2), as well as under some additional bosonic transformations, whose role is to provide on-shell closure of the full algebra. The action presented includes a space-like vector n^μ as an auxilliary variable, which turns out to be gauged away. (On this reason, it is not necessary to consider a pair of superstrings in our construction.) Since the variable n^μ is treated on equal footing with other ones, the symmetry transformations form a superalgebra in the usual sence (without occurence of nonlinear in generator terms in the right hand side of Eq. (1)), in contrast to Refs. 7, 12, 13. Further, one possible gauge is $n^\mu = (0, \ldots, 0, 1)$. In this gauge Eq. (2) reduces (in

our Γ-matrix notations [14]) to

$$\begin{aligned}
&\delta\theta_\alpha = \epsilon^\alpha, \qquad \delta\bar{\theta}_\alpha = \bar{\epsilon}^\alpha, \\
&\delta x^{\bar{\mu}} = -i\bar{\epsilon}_\alpha\tilde{\Gamma}^{\bar{\mu}\alpha\beta}\bar{\theta}_\beta - i\epsilon^\alpha\Gamma^{\bar{\mu}}_{\alpha\beta}\theta^\beta, \qquad \delta x^{11} = 0,
\end{aligned} \tag{3}$$

where $\theta = (\bar{\theta}_\alpha, \theta^\alpha)$, $\mu = (\bar{\mu}, 11)$, $\bar{\mu} = 0, 1, \ldots, 9$, $\alpha = 1, \ldots, 16$. Eq. (3) exactly coincides with the standard $D = 10, N = 2$ supersymmetry transformations. Thus, in our case one can treat the new supersymmetry (2) as a way to rewrite the $D = 10, N = 2$ supersymmetry in the "eleven dimensional notations".

The work is organized as follows. In Sec. 2 we present and discuss a model of a nondynamical space-like vector n^μ, which seems to be a necessary part of our construction suggested in the next Section. In Sec. 3 covariant action for closed $D = 11$ superstring is suggested. Its global symmetries are founded and prove to form an on-shell closed algebra. Generalized local κ-symmetry is also presented. In Sec. 4 within the Hamiltonian framework it is shown that physical sector variables and their dynamics coincide with those of the $D = 10$ type IIA Green–Schwarz (GS) superstring [15]. Thus, one gets the corresponding supersymmetric spectrum on the quantum level. In Sec. 5 it is demonstrated that the action for $D = 10$ type IIA GS superstring can be considered as a partially gauge fixed version of the $D = 11$ superstring action.

2 Action for a nondynamical space-like vector

As was mentioned in the Introduction, we need to get in our disposal a nondynamical space-like vector field. So, as a preliminary step of our construction, let us discuss the following $D = 11$ Poincaré invariant action

$$S = -\int d^2\sigma \left[n^\mu \varepsilon^{ab} \partial_a A^\mu_b + \frac{1}{\phi}(n^2 + 1) \right], \tag{4}$$

which turns out to be a building block of the eleven dimensional superstring action considered below. Here $n^\mu(\sigma^a)$ is $D=11$ vector and $d=2$ scalar, $A^\mu_a(\sigma^b)$ is $D=11$ and $d=2$ vector, while $\phi(\sigma^a)$ is a scalar field. In Eq. (4) we have set $\varepsilon^{ab}=-\varepsilon^{ba}$, $\varepsilon^{01}=-1$, $\eta_{\mu\nu}=(+,-,\ldots,-)$ and it is also supposed that $\sigma^1\subset[0,\pi]$. From the equation of motion $\delta S/\delta\phi=0$ it follows that n^μ is a space-like vector.

Local symmetries of the action are the $d=2$ reparametrizations[1] and the following transformations with the parameters $\rho^\mu(\sigma^a)$, $\omega_a(\sigma^b)$

$$\begin{aligned}\delta A^\mu_a&=\partial_a\rho^\mu+\omega_a n^\mu;\\ \delta\phi&=\frac{1}{2}\phi^2\varepsilon^{ab}\partial_a\omega_b.\end{aligned}\tag{5}$$

These symmetries are reducible because their combination with the parameters of a special form: $\omega_a=\partial_a\omega$, $\rho^\mu=-\omega n^\mu$ is a trivial symmetry: $\delta_\omega A^\mu_a=-\omega\partial_a n^\mu$, $\delta_\omega\phi=0$ (note that $\partial_a n^\mu=0$ is one of the equations of motion). Thus, Eq. (5) includes 12 essential parameters which correspond to the primary first class constraints $p^\mu_0\approx 0$, $\pi_\phi\approx 0$ in the Hamilton formalism (see below).

Let me demonstrate a nondynamical character of the model. For this aim the Hamiltonian formalism seems to be the most appropriate, since second class constraints must be taken into account. Momenta conjugate to the variables n^μ, A^μ_a, ϕ are denoted by p^μ_n, p^μ_a, π_ϕ. All equations for determining the momenta turn out to be the primary constraints

$$\pi_\phi=0,\qquad p^\mu_0=0;\tag{6}$$

$$p^\mu_n=0,\qquad p^\mu_1-n^\mu=0.\tag{7}$$

[1]Note that interaction with the $d=2$ metric $g^{ab}(\sigma^c)$ is not necessary due to the presence of ε^{ab} symbol and the supposition that the variable ϕ transforms as a density $\phi'(\sigma')=\det(\partial\sigma'/\partial\sigma)\phi(\sigma)$ under the reparametrizations.

The canonical Hamiltonian is

$$H = \int d\sigma^1 \left[n^\mu \partial_1 A_0^\mu + \frac{1}{\phi}(n^2+1) + \lambda_\phi \pi_\phi + \lambda_n^\mu p_n^\mu + \lambda_0^\mu p_0^\mu + \lambda_1^\mu (p_1^\mu - n^\mu) \right], \tag{8}$$

where λ_* are the Lagrange multipliers corresponding to the constraints. The preservation in time of the primary constraints implies the secondary ones

$$\partial_1 n^\mu = 0, \qquad n^2 + 1 = 0, \tag{9}$$

and equations for determining some of the Lagrange multipliers

$$\lambda_1^\mu = \partial_1 A_0^\mu + \frac{2}{\phi} n^\mu, \qquad \lambda_n^\mu = 0. \tag{10}$$

The tertiary constraints are absent.

Constraints (7) form a system of second class and can be omitted after introducing the corresponding Dirac bracket (the Dirac brackets for the remaining variables coincide with the Poisson ones). After imposing the gauge fixing conditions $\phi = 2$, $A_0^\mu = 0$ for the first class constraints (6), dynamics of the remaining variables is ruled by the equations

$$\dot{A}_1^\mu = p_1^\mu, \qquad \dot{p}_1^\mu = 0, \qquad (p_1^\mu)^2 = -1, \qquad \partial_1 p_1^\mu = 0. \tag{11}$$

Then the gauge conditions $A_1^{11} = \tau$, $A_1^{\bar{\mu}} = 0$ are selfconsistent and lead to $p_1^{11} = 1$, $p_1^{\bar{\mu}} = 0$. It should be stressed also that the equation $p_1^{11} = 1$ is consistent with the closed string boundary conditions only [14]. Hence, the model (4) is selfconsistent being considered on the closed world sheet only.

Thus, we have demonstrated that one of the possible gauges to the theory (4) is

$$A_1^{11} = \tau, \qquad p_1^{11} = n^{11} = 1, \qquad \phi = 2, \tag{12}$$

with all other variables vanishing. Adding of this action to any model is one of the ways to introduce (without change of the initial dynamics) a space-like vector n^μ, that may further be used as appropriate. The action of such a kind was successfully used before [16] in a different context.

3 Action of $D = 11$ superstring and its symmetries

$D = 11$ superstring action to be examined is of the form

$$S = \int d^2\sigma \left\{ \frac{-g^{ab}}{2\sqrt{-g}} \Pi_a^\mu \Pi_{b\mu} - i\varepsilon^{ab}(\partial_a x^\mu - \frac{i}{2}\bar{\theta}\Gamma^{\mu\nu} n_\nu \partial_a \theta)(\bar{\theta}\Gamma_\mu \partial_b \theta) - \right.$$
$$\left. -\varepsilon^{ab}\xi_a(n_\mu \Pi_b^\mu) - n^\mu \varepsilon^{ab}\partial_a A_b^\mu - \frac{1}{\phi}(n^2+1) \right\}, \qquad (13)$$

where θ is a 32-component Majorana spinor of $SO(1,10)$, ξ_a is a $d=2$ vector and it was denoted $\Pi_a^\mu \equiv \partial_a x^\mu - i\bar{\theta}\Gamma^{\mu\nu} n_\nu \partial_a \theta$. The meaning of the last two terms was explained in the previous section. The third term is crucial for existence of local κ-symmetry and, at the same time, it provides the split of the x^{11} coordinate from the physical sector (see below).

Let me describe global symmetries structure of the action (13). Bosonic symmetries are the $D = 11$ Poincaré transformations in the standard realization and the following ones with antisymmetric parameter $b^{\mu\nu} = -b^{\nu\mu}$:

$$\begin{aligned} &\delta_b x^\mu = b^\mu{}_\nu n^\nu, \\ &\delta_b A_a^\mu = -b^\mu{}_\nu \left(\varepsilon_{ab} \frac{g^{bc}}{\sqrt{-g}} \Pi_c{}^\nu - \xi_a n^\mu + i(\bar{\theta}\Gamma^\nu \partial_a \theta) \right). \end{aligned} \qquad (14)$$

There are also fermionic supersymmetry transformations being realized as follows:

$$\begin{aligned} &\delta\theta = \epsilon, \\ &\delta x^\mu = i\bar{\epsilon}\Gamma^{\mu\nu} n_\nu \theta, \\ &\delta A_a^\mu = i\varepsilon_{ab}\frac{g^{bc}}{\sqrt{-g}}\Pi_{c\nu}(\bar{\epsilon}\Gamma^{\mu\nu}\theta) - \frac{5}{6}(\bar{\epsilon}\Gamma^{\nu\mu}\theta)(\bar{\theta}\Gamma_\nu \partial_a \theta) + \\ &\qquad + \frac{1}{6}(\bar{\epsilon}\Gamma_\nu \theta)(\bar{\theta}\Gamma^{\nu\mu}\partial_a \theta). \end{aligned} \qquad (15)$$

One can prove that the complete algebra is on-shell closed up to the equation of motion $\partial_a n^\mu = 0$ and trivial transformations $\delta A_a^\mu = \partial_a \rho^\mu$ (see

Eq. (5)) with field-dependent parameter ρ^μ, as it usually happens in component formulations of supersymmetric models without auxilliary fields. The only nontrivial commutator is[2]

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_b, \qquad b^{\mu\nu} = -2i(\bar{\epsilon}_1 \Gamma^{\mu\nu} \epsilon_2). \tag{16}$$

Let me note that one needs to use the $D = 11$ Fierz identities to prove Eq. (16) for A_a^μ variable

$$(C\Gamma^\mu)_{\alpha(\beta}(C\Gamma^{\mu\nu})_{\gamma\delta)} + (C\Gamma^{\mu\nu})_{\alpha(\beta}(C\Gamma^\mu)_{\gamma\delta)} = 0. \tag{17}$$

The relation of Eq. (15) to the $D = 10, N = 2$ supersymmetry has been described in the Introduction.

Local bosonic symmetries for the action (13) are the $d = 2$ reparametrizations (with the standard transformation lows for all variables except the variable ϕ, which transforms as a density: $\phi'(\sigma') = \det(\partial\sigma'/\partial\sigma)\phi(\sigma)$), Weyl symmetry, and the transformations with parameters $\rho^\mu(\sigma^a)$ and $\omega_a(\sigma^b)$ described in the previous Section.

The action is also invariant under a pair of local fermionic κ-symmetries. To find them, let me consider the following ansatz:

$$\begin{aligned} \delta\theta &= \pm \Pi_{d\mu} S^\pm \Gamma^\mu \kappa^{\mp d}, \\ \delta x^\mu &= -\delta\bar{\theta}\Gamma^{\mu\nu} n_\nu \theta, \\ \delta g^{ab} &= 8i\sqrt{-g} P^{\pm ca}(\partial_c \bar{\theta} S^\mp \kappa^{\mp b}), \end{aligned} \tag{18}$$

where it was denoted

$$S^\pm = \frac{1}{2}(1 \pm n_\mu \Gamma^\mu), \quad \kappa^{\mp d} \equiv P^{\mp dc}\kappa_c, \quad P^{\mp dc} = \frac{1}{2}\left(\frac{g^{dc}}{\sqrt{-g}} \mp \varepsilon^{dc}\right). \tag{19}$$

[2]To elucidate relation between Eqs. (16) and (1) let me point a simple analogy: algebra of the Lorentz generators $M^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu$ can be written either as $[M^{\mu\nu}, M^{\rho\sigma}] = \eta^{\mu\rho} M^{\nu\sigma} + \ldots$ or $[M^{\mu\nu}, M^{\rho\sigma}] = -\eta^{\mu\rho} p^\sigma x^\nu + \ldots$. The second case may be considered as corresponding to Eq. (1).

Note that on-shell (where $n^2 = -1$) the $S^{\pm}{}_{\alpha}{}^{\beta}$ – operators form a pair of projectors in θ-space. Let me recall also that the $d = 2$ projectors $P^{\pm}$ obey the following properties: $P^{+ab} = P^{-ba}$, $P^{\mp ab}P^{\mp cd} = P^{\mp cb}P^{\mp ad}$.

After tedious calculations with the use of these properties and the Fierz identities (17), a variation of the action (13) under the transformations (18) can be presented in the form

$$\delta S = -\varepsilon^{ab}\partial_a n_\nu G_b^\nu - \frac{1}{\phi^2}(n^2+1)H + \varepsilon^{ab}(n_\mu \Pi_b^\mu)F_a, \tag{20}$$

where

$$\begin{aligned} G_b{}^\nu \equiv & -i\varepsilon_{bc}\frac{g^{cd}}{\sqrt{-g}}(\delta\bar\theta\Gamma^{\mu\nu}\theta)\Pi_{d\mu} + \frac{1}{2}(\delta\bar\theta\Gamma^{\mu\nu}\theta)(\bar\theta\Gamma_\mu\theta) - \\ & -\frac{1}{2}(\delta\bar\theta\Gamma_\mu\theta)(\bar\theta\Gamma^{\mu\nu}\partial_b\theta) + i\xi_b(\delta\bar\theta\Gamma^{\mu\nu}\theta)n_\nu, \\ H \equiv & -i\phi^2\frac{g^{ab}}{\sqrt{-g}}(\partial_a\bar\theta\Gamma^\mu\tilde\kappa^\mp)\Pi_{b\mu}, \\ F_a \equiv & \, i[\varepsilon_{ac}\frac{g^{cd}}{\sqrt{-g}}(\partial_d\bar\theta\Gamma^\mu\tilde\kappa^\mp)n_\mu + (\partial_a\bar\theta\tilde\kappa^\mp)\mp \\ & \mp 2\varepsilon_{ab}P^{\pm\,cd}(\partial_c\bar\theta\Gamma^\mu\kappa^{\mp\,b})\Pi_{d\mu}], \end{aligned} \tag{21}$$

and it was denoted $\tilde\kappa^\mp \equiv \Pi_{a\mu}\Gamma^\mu\kappa^{\mp\,a}$. All the terms in Eq. (20) can evidently be cancelled by corresponding variations of the auxiliary fields

$$\delta A_b{}^\nu = G_b{}^\nu, \qquad \delta\phi = H, \qquad \delta\xi_a = F_a. \tag{22}$$

In the result, eleven dimensional superstring action (13) is invariant under transformations from Eq. (18) supplemented by ones from Eq. (22). Let me stress that all three last terms in the action turn out to be essential for achieving this local κ-symmetry.

Since in Eq. (18) there appeared the double projectors ($S^{\pm}$ and $\Pi_{a\mu}\Gamma^\mu$) acting on the θ-space, the total number of essential parameters is $8+8$. Their relation with the $D = 10, N = 2$ GS superstring κ-symmetry will be described in the last Section.

4 Analysis of dynamics

The aim of this Section is to demonstrate that physical variables of the theory (13) and their dynamics exactly coincide with those of the $D = 10$ type IIA GS superstring [15].

Following the standard Hamiltonian procedure one finds a pair of second class constraints ${p_n}^\mu = 0$, ${p_1}^\mu - n^\mu = 0$ among primary constraints of the theory. Then variables $(n^\mu, {p_n}^\mu)$ can be omitted after introducing the associated Dirac bracket (see Sec. 2). The Dirac brackets for the remaining variables coincide with the Poisson ones, and the Hamiltonian with primary constraints then looks like

$$H = \int d\sigma^1 \left\{ -\frac{N}{2}(\hat{p}^2 + \Pi_{1\mu}\Pi_1^\mu) - N_1 \hat{p}_\mu \Pi_1^\mu + p_{1\mu}\partial_1 A_0^\mu - \xi_0(p_{1\mu}\partial_1 x^\mu) + \right.$$
$$\left. + \frac{1}{\phi}(p_1^2 + 1) + \lambda_\phi \pi_\phi + \lambda_{0\mu} p_0^\mu + \lambda^{ab}(\pi_g)_{ab} + \lambda_{\xi a} {p_\xi}^a + L_\alpha {\lambda_\theta}^\alpha \right\}, \qquad (23)$$

where p^μ, p_0^μ, p_1^μ, $p_{\xi a}$, $(\pi_g)_{ab}$ are momenta conjugate to the variables x^μ, A_0^μ, A_1^μ, ξ_a, g_{ab}, respectively; λ_* are Lagrange multipliers corresponding to the primary constraints. In Eq. (23) we also denoted

$$N = \frac{\sqrt{-g}}{g^{00}}, \quad N_1 = \frac{g^{01}}{g^{00}}, \quad \hat{p}^\mu = p^\mu - i\bar{\theta}\Gamma^\mu\partial_1\theta + \xi_1 p_1^\mu,$$
$$L_\alpha \equiv p_{\theta\alpha} - i(p^\mu - \frac{i}{2}\bar{\theta}\Gamma_\mu\partial_1\theta)\bar{\theta}\Gamma^{\mu\nu}p_{1\nu} - i(\partial_1 x^\mu - \frac{i}{2}\bar{\theta}\Gamma^{\mu\nu}p_{1\nu}\partial_1\theta)\bar{\theta}\Gamma_\mu = 0. \qquad (24)$$

The full system of constraints can be presented in the form

$$(\pi_g)_{ab} = 0, \qquad \pi_\phi = 0, \qquad p_{\xi a} = 0, \qquad p_0^\mu = 0; \qquad (25.a)$$

$$\partial_1 p_1^\mu = 0, \qquad (p_1^\mu)^2 = -1; \qquad (25.b)$$

$$\hat{p}^\mu p_{1\mu} = 0, \qquad \partial_1 x^\mu p_{1\mu} = 0; \qquad (25.c)$$

$$(\hat{p}^\mu \pm \Pi_1^\mu)^2 = 0, \qquad L_\alpha = 0. \qquad (25.d)$$

Besides, some of the Lagrange multipliers have been determined in the process

$$\lambda_n^\mu = 0, \qquad \lambda_{A1}^\mu = \partial_1 A_0^\mu + \frac{2}{\phi} p_1^\mu + Q^\mu, \tag{26}$$

where

$$Q^\mu \equiv -N\xi_1 \hat{p}^\mu + N_1\xi_1 \Pi_1^\mu - \xi_0 \partial_1 x^\mu + \frac{1}{2}[(\bar\theta\Gamma_\nu\partial_1\theta)\bar\theta\Gamma^{\mu\nu} + (\bar\theta\Gamma^{\mu\nu}\partial_1\theta)\bar\theta\Gamma_\nu]\lambda_\theta. \tag{27}$$

To go further let me impose gauge fixing conditions to the first class constraints (25.a). The choise consistent with the equations of motion is

$$\begin{gathered} g^{ab} = \eta^{ab}, \qquad \phi = 2, \qquad \xi_a = 0, \\ A_0^\mu = -\int_0^\sigma d\sigma' Q^\mu(\sigma'), \end{gathered} \tag{28}$$

where Q^μ is given by Eq. (27). This choise for A_0^μ simplifies subsequent analysis of the A_1^μ, p_1^μ sector. Namely, dynamics of these variables is ruled now by the equations

$$\partial_0 A_1^\mu = p_1^\mu, \qquad \partial_0 p_1^\mu = 0, \tag{29}$$

and by the first class constraints (25.b). The following gauge: $A_1^{11} = \tau$, $A_1^{\bar\mu} = 0$, can be imposed, which breaks manifest $SO(1,10)$ covariance up to $SO(1,9)$ one. One gets also $p_1^{11} = 1$, $p_1^{\bar\mu} = 0$ and the constraints (25.c) are reduced to $\hat{p}^{11} = 0$, $\partial_1 x^{11} = 0$. These are a pair of second class constraints which simply mean that the variables (x^{11}, p^{11}) can now be omitted.

In the result we stay with the situation of the $D = 10$ GS superstring (see Eq. (25.d)), and the subsequent analysis coincides with that well known case [17, 18]. Namely, physical variables sector contains the transverse components x^i, $i = 1, \ldots, 8$, of the coordinate $x^{\bar\mu}$, $\bar\mu = 0, 1, \ldots, 9$, and a pair of $SO(8)$ spinors of opposite chirality $(\bar\theta_{\dot a}, \theta_a)$, $a, \dot a = 1, \ldots, 8$.

They are related to the initial θ-variable as follows:

$$\theta = \begin{pmatrix} \bar\theta_\alpha \\ \theta_\alpha \end{pmatrix}, \quad \alpha = 1, \ldots, 16, \quad \theta^\alpha = \begin{pmatrix} S_a \\ \bar\theta_{\dot a} \end{pmatrix}, \quad \bar\theta^\alpha = \begin{pmatrix} \theta_a \\ \bar S_{\dot a} \end{pmatrix}. \tag{30}$$

Dynamics of the physical variables

$$\begin{aligned} &\partial_0 x^i = -p^i, \qquad \partial_0 p^i = -\partial_1\partial_1 x^i, \\ &(\partial_0+\partial_1)\theta_a = 0, \qquad (\partial_0-\partial_1)\bar\theta_{\dot a} = 0, \end{aligned} \tag{31}$$

as well as quantum states spectrum of the $D=11$ superstring (13) exactly coincide with those of the $D=10$ type IIA GS superstring.

5 Reduction to $D=10$ type IIA GS superstring action

Type IIA string action can be considered as partially gauge fixed version of the $D=11$ superstring (13), where $SO(1,10)$ invariance is broken up to $SO(1,9)$. To demonstrate this, let me substitute the gauge $n^\mu = (0,\ldots,0,1)$, $\xi_a = 0$ in Eq. (13) (then equations of motion for ξ_a-variable mean that $\partial_a x^{11} = 0$). By using of $SO(1,9)$ decomposition for $SO(1,10)$ objects [14]

$$\begin{aligned} &\Gamma^\mu = \{\Gamma^{\bar\mu}, \Gamma^{11}\} = \left\{ \begin{pmatrix} 0 & \Gamma^{\bar\mu} \\ \tilde\Gamma^{\bar\mu} & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}, \\ &\theta = (\bar\theta_\alpha, \theta^\alpha), \qquad S^+ = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \qquad S^- = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \\ &\qquad \{\Gamma^{\bar\mu}, \tilde\Gamma^{\bar\nu}\} = -2\eta^{\bar\mu\bar\nu}, \qquad \eta^{\bar\mu\bar\nu} = (+,-,\ldots,-), \\ &\qquad \alpha = 1,\ldots,16, \qquad \bar\mu = 0,1,\ldots,9, \\ &\qquad \bar\theta\Gamma^{\mu\nu}n_\nu\psi = -\theta\Gamma^{\bar\mu}\psi - \bar\theta\tilde\Gamma^{\bar\mu}\bar\psi, \\ &\qquad \bar\theta\Gamma^\mu\psi = \{\bar\theta\tilde\Gamma^{\bar\mu}\bar\psi - \theta\Gamma^{\bar\mu}\psi;\ -\theta\bar\psi - \bar\theta\psi\}, \end{aligned} \tag{32}$$

the resulting expression can be rewritten (up to total derivative) in the form

$$S = \int d^2\sigma \Big\{ \frac{-g^{ab}}{2\sqrt{-g}}[\partial_a x^{\bar\mu} + i(\theta\Gamma^{\bar\mu}\partial_a\theta) + i\bar\theta\tilde\Gamma^{\bar\mu}\partial_a\bar\theta]^2 -$$

$$-i\varepsilon^{ab}\partial_a x^{\bar\mu}(\bar\theta\tilde\Gamma^{\bar\mu}\partial_b\bar\theta - \theta\Gamma^{\bar\mu}\partial_b\theta) + \varepsilon^{ab}(\theta\Gamma^{\bar\mu}\partial_a\theta)(\bar\theta\tilde\Gamma^{\bar\mu}\partial_b\bar\theta)\Big\}, \qquad (33)$$

which coincides with type IIA GS action [15]. In the similar fashion, global supersymmetry transformations (15) reduces to the standard $N = 2$ supersymmetry (3), as it was mentioned in the Introduction. At last, by using of Eq. (32) the generalized $D = 11$ κ-symmetry (18) reduces to $D = 10$ Siegel κ-symmetry of GS superstring action

$$\begin{aligned}
&\delta\theta^\alpha = -P^{-cd}\Pi^{\bar\mu}{}_d\tilde\Gamma^{\bar\mu\,\alpha\beta}\bar\kappa_{c\beta}, \qquad \delta\bar\theta^\alpha = P^{+cd}\Pi^{\bar\mu}{}_d\Gamma^{\bar\mu}_{\alpha\beta}\kappa_c{}^\beta,\\
&\delta x^\mu = i\theta^\alpha\Gamma^{\bar\mu}_{\alpha\beta}\delta\theta^\beta + i\bar\theta_\alpha\tilde\Gamma^{\bar\mu\,\alpha\beta}\delta\bar\theta^\beta,\\
&\delta g^{ab} = 8i\sqrt{-g}\{P^{-ca}(\partial_c\bar\theta\kappa^{+b}) - P^{+ca}(\partial_c\theta\bar\kappa^{-b})\}.
\end{aligned} \qquad (34)$$

In conclusion, in this letter $D = 11$ Poincaré invariant superstring action based on the new superslgebra (14)–(16) different from the super Poincaré one was suggested. Physical sector variables, their dynamics and states spectrum of the model coincide with one for the $D = 10$ type IIA GS superstring. In accordance with the results of Refs. 7 and 13 one expects critical dimension of the theory is $D = 11$. One may hope that similar construction will works for lifting of the $D = 10$ type IIB string to corresponding (10,2) version (see also Ref. 13). It will be interesting also to apply the scheme developed in this work for construction of Lagrangean formulation for $(D-2,2)$ SYM equations of motion considered in Refs. 9, 10. Note also that algebra of supersymmetry transformations is closed on-shell only, and an intriguing problem is to find a formulation with off-shell closed version of the superalgebra.

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